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A Discussion of Programmatic Differences Within Mathematics Content Courses for Prospective Elementary Teachers

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Prospective elementary teacher (PSET) education programs vary greatly in the courses and course sequences employed to prepare their students. This article explores potential tradeoffs that arise for mathematics teacher educators, PSETs, and their future students due to the choices PSET education programs make regarding their design. Specifically, the sequencing of content and pedagogy across courses, integration of content and pedagogy within courses, content coverage, mathematical rigor, and interactions between PSETs' beliefs and experiences are explored from the perspective of mathematics teacher educators using vignettes. Based on the vignettes and literature, future directions for research regarding PSET education program design are suggested.

Elementary school teachers play a crucial role in laying the initial groundwork for children's mathematical thinking and reasoning (Association of Mathematics Teacher Educators [AMTE], 2017). They also play a central role in creating a supportive learning environment for children to develop positive dispositions towards mathematics (National Council of Teachers of Mathematics [NCTM], 2000). As such, preparing prospective

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elementary teachers for these vital roles is an important task for mathematics teacher educators (MTEs).

Policy documents such as the Conference Board of the Mathematical Sciences' (CBMS; 2012) *Mathematical Education of Teachers II* (MET II) and the AMTE (2017) *Standards for the Preparation of Teachers of Mathematics* (SPTM) provide guidance as to what elementary teachers should know and be able to do to teach mathematics. However, standards only provide a benchmark as to where we want to be at the end of a process (Goertz, 2010). The problem is, then, how do we achieve those benchmarks? In other words, how do we prepare prospective elementary teachers (PSETs) to teach mathematics with those end goals in mind? This is a question the authors grapple with as MTEs responsible for teaching mathematics content courses to prepare PSETs.

On the other hand, we acknowledge that, just as our K–12 education system is best described as diverse and highly variable from context to context (Schmidt & McKnight, 2012; Schmidt et al., 2005; Sosina, 2020), the same could be said of the preparation of mathematics teachers. A national survey of teacher preparation programs found, "the majority (56.7%) of schools having mathematics content courses specifically for prospective elementary teachers offer two of these mathematics content courses, while 17.1% offer three, 16.1% offer one, and 9.9% offer from four to 12 of these courses" (Masingila et al., 2012, p. 352). In addition to the variety in teacher preparation programs, MTEs hold varying beliefs as to the goals of PSET education programs. These differing goals lead to a variety of often contrasting approaches in the enactment of instruction designed to meet these aims.

Our goal for this paper is not to answer definitively the aforementioned questions about how to teach PSETs, but to foster discussion in the MTE community and to highlight the need for more research considering those questions, specifically about mathematics content courses. In our effort to achieve this goal, we present three fictitious vignettes based on our experiences and those of others in our positions. The purposes of the vignettes are two-fold. First, we intend to demonstrate issues experienced by instructors of mathematics content courses for PSETs. Second, we mean to provide examples of the variability in the preparation of PSETs. The vignettes are followed by a discussion to highlight some important issues we believe merit further investigation.

Vignette 1: The Issue of Sequencing and Integrating Content and Pedagogy

Dr. Browne teaches at a university that has an elementary education program that is housed within its education department. However, every student in the program is required to take one mathematics content course specifically designed for PSETs, no matter their content concentration, taught by faculty members in the mathematics department, like Dr. Browne. The course is designed to accomplish several goals: teach content that is horizon knowledge for PSETs; engage PSETs in mathematical problem solving; foster positive and productive mathematical identities; and model good pedagogy for teaching mathematics. The course content is focused on algebraic and statistical reasoning and problem solving. This course is the first mathematics-focused course that PSETs take in their program, and they will later take additional mathematics pedagogy courses from faculty in the education department focused on teaching elementary mathematics content (e.g., number and operation, geometry, measurement, and operations and algebraic thinking).

Based on the sequence of the courses and their home departments, Dr. Browne has experienced tension in how much pedagogical instruction should be presented in the content course. Because the course is a content course, the main goal is teaching mathematics content. However, the course is only for PSETs, so issues of pedagogy seem relevant and important for all students. As Ball et al. (2005) pointed out, “knowing mathematics for teaching demands a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably” (p. 22). This body of teachers’ knowledge impacts students’ achievement (Kutaka et al., 2017).

Believing that PSETs exposed to good pedagogy earlier in their programs develop deeper pedagogical content knowledge, Dr. Browne desires to explicitly model research-based pedagogy and provide her pedagogical philosophies and rationales for making certain pedagogical moves during her teaching. She also incorporates readings on mathematics pedagogy and gives students assignments related to the readings, such as sharing ideas in online discussions or writing lesson plans that model the pedagogies they have read. However, she has encountered colleagues who worry that this approach takes away from the mathematical rigor of the course. Some faculty members believe strongly that the course is a mathematics course, and should focus solely on content, leaving pedagogical discussions as the responsibility of the faculty in the education department.

Where do discussions of pedagogy belong? Should these discussions occur in content courses, so students can begin to think like teachers from the beginning of their teacher preparation program, or in pedagogy courses, once students have demonstrated mastery of the content?

Vignette 2: The Tension in Determining Content Coverage and Mathematical Rigor

At a recent conference, two colleagues met for coffee to catch up. As often happens, the conversation included discussion of their respective research projects and teaching responsibilities. Dr. Mathews is employed as a mathematics education professor within a department of mathematics, while Dr. Eduardo is employed as a mathematics education professor within a department of education. Both are assigned to teach mathematics content courses for PSETs. Through the course of their conversation, the two colleagues noticed many substantive differences in the way content courses are expected to cover content and involve mathematical rigor at their respective institutions.

Within the mathematics department where Dr. Mathews is employed, the focus is on exposing PSETs to as much content as possible, as rigorously as possible, within the allotted time. Discussions of pedagogy, including common misconceptions elementary students may have and the effective use of manipulatives to teach elementary content, are viewed as topics best left for pedagogy courses. Instead, course time is spent reintroducing (or in some cases, introducing) PSETs to wide ranging topics, including, but not limited to: set theory; logic; number theory; whole number, decimal, and fraction operations; algebra and functions; statistics and probability; and geometry and measurement. The driving philosophy is that, in order to effectively teach elementary content, the PSETs must learn material that far exceeds what they will teach.

Relatedly, Dr. Mathews has experienced difficulties in motivating PSETs to learn mathematics topics not directly related to the content standards they will have to teach. This view is shared by some faculty in the education department, who wonder why PSETs take an entire course on content outside of the specific content they will be expected to teach. Dr. Mathews has tried to address this by making explicit connections between the content and the elementary mathematics standards whenever possible. For example, in her teaching about set theory, she helps PSETs discover the mathematical structure and meaning involved in the intersection of the set of rhombuses and the set of rectangles being the set of squares. This directly relates to a third grade content standard (Common Core State Standards Initiative, 2010). However, given the mathematical content she is required to cover, it is not easy to build in this time for every topic.

Within the department of education where Dr. Eduardo teaches, the focus is on blending mathematical content knowledge with pedagogical content knowledge. While limiting the PSETs' exposure to a few foundational topics, such as whole number operations, students are expected to exhibit deep conceptual understanding. In this setting, deep understanding consists of developing the meaning behind the mathematics, the ability to solve problems in multiple ways, and the knowledge and anticipation of common misconceptions elementary students may encounter. Fewer topics are covered than in the courses taught by Dr. Mathews, and indeed, some topics included in the *Common Core State Standards for Mathematics* (CCSS-M; Common Core State Standards Initiative, 2010) are not covered at all. Relatedly, Dr. Eduardo has experienced pushback, especially from PSETs, that they were not exposed to all the content they could teach in the future. The subsequent methods courses then focus on the previously absent pedagogical strategies, such as questioning techniques and lesson planning, with the assumption that the mathematics is prerequisite knowledge.

As their conversation progresses and their coffee cups empty, Dr. Mathews and Dr. Eduardo are left wondering which model best serves their PSETs in preparing them to effectively teach elementary mathematics. Should PSETs be exposed to as much of the standards as they will be expected to teach? Or, it is more beneficial to cover only a subset of the standards, but do so in great depth? What should mathematically rigorous content coverage look like in courses such as this? While the two professors did not reach consensus on these questions, they concluded their conversation with greater appreciation for the affordances and limitations of each model, and mutually agreed to continue this important debate throughout the year and at next year's conference.

Vignette 3: Conflicting Views of Mathematics, Learning, and Teaching

Dr. Tusi was trained as an MTE in a college of education and had experience teaching pedagogy courses during his graduate studies. Dr. Tusi is now in a mathematics department in which he is expected to teach content courses for PSETs, who have not yet taken any pedagogy courses in their program. During the fall semester, Dr. Tusi taught two sections of a geometry content course, the second content course these PSETs were required to take in the math department, before taking a pedagogy course in the department of education. In this course, Dr. Tusi learned many of his students could recall definitions or formulas from their past experience, so he focused instead on discussing why the formulas are valid or constructing connections between various topics. In other words, in addition to having procedural fluency, Dr. Tusi wanted his students to have conceptual understanding (AMTE, 2017; Hodgson, 2001; National Research Council, 2001; NCTM, 2014). Dr. Tusi believes PSETs should be prepared not only to "know" or "solve" a mathematical problem, but also to engage in discussions with students, which involves making sense of students' thinking, entertaining different ways of thinking, and responding and accommodating appropriately in the moment (Loewenberg Ball et al., 2008; NCTM, 2012). Therefore, he found it was important to present his PSETs with samples of children's work to discuss ways that children might think about these topics, which could be very different from the ways PSETs think about the mathematics. Such an approach has been documented to positively impact PSETs' mathematical content knowledge and beliefs (Philipp et al., 2007).

As the semester came to an end, Dr. Tusi reflected on teaching this content course. He experienced tensions when making decisions about the scope and rigor of the content and the extent to which he should include pedagogical considerations in his course, similar to those discussed in the previous two vignettes. In addition, there were other tensions he grappled with throughout teaching the course. The PSETs brought in varying views about doing mathematics and how it should be learned (CBMS, 2012), along with varying perceptions of themselves in relation to mathematics and as teachers. Dr. Tusi noticed that the views and perceptions of selves PSETs brought into his classroom interacted with his own beliefs, such as valuing conceptual understanding and students' mathematical thinking.

For example, during the third class into the semester, while solving a problem on angle measure, a PSET in his class raised her hand and questioned the goal of the course. The PSET elaborated on her question, explaining she was confused because the topics were those she had already learned from middle school geometry, she felt she already knew them well, and she would not teach all of these topics at the elementary level. Dr. Tusi took this as an opportunity to reiterate the goal of the course and emphasized that knowing a concept or how to solve a problem independently is very different from being able to teach it to someone who may think differently. Therefore, it is important for PSETs to articulate explanations of concepts and to think about how children might approach them.

While many of his PSETs got on board with this idea, Dr. Tusi also noticed his PSETs brought in differing views on how mathematics should be learned. These varying views were accentuated between the two sections of the course, as the PSETs in each section demonstrated different dispositions toward the way the course was taught. The PSETs in the first section generally demonstrated an enthusiasm towards understanding the "why" behind concepts or formulas and found value in discussing children's sample work. They attempted to make sense of the different ways children could think about a mathematical topic and how they might attend to these differences as a teacher. Although this approach to learning and doing math was different from their past experiences, these PSETs appreciated it. On the other hand, some PSETs in the second section demonstrated discomfort with the way the course was taught. They resisted an approach focused on conceptual understanding, instead favoring a more traditional approach where correct answers are the sole objective.

While the PSET who raised the question about the goal of the course was confident about her mathematical understandings, Dr. Tusi noticed there were also PSETs who either expressed a dislike towards mathematics, or, more devastatingly, believed they were incapable of doing mathematics. Oftentimes, he heard PSETs prefacing their questions or solutions with comments such as, "I know this is a dumb question," "Everyone else probably already knows this," "I am just not a visual person," or "I've never been good at math." Although he addressed such comments in the moment at a surface level to encourage students to engage in productive struggle (Hiebert & Grouws, 2007) and develop a growth mindset (Boaler, 2016), Dr. Tusi felt like such views were learned and accumulated over a long time and he wished he knew better ways to support his PSETs in the context of a content course. He was especially concerned about these views from an equity standpoint. Most of his students would soon begin their careers as teachers, and it is well documented that elementary students "with the greatest needs are often taught by teachers with the least experience" (AMTE, 2017, p. 7).

Therefore, Dr. Tusi felt it was crucial to attempt to adjust these dispositions toward mathematics.

Dr. Tusi wanted to explicitly address his PSETs' varying views about doing mathematics and how it should be learned and taught in his course. He also wanted to better address his PSETs' varying perceptions of themselves in relation to mathematics. However, he felt the tension between having to cover content in a limited time and addressing important implications of such beliefs in his content course.

Issues Raised by Vignettes

In this section, we discuss the major issues raised in each vignette including: (a) sequencing content and pedagogy across courses, (b) integrating content and pedagogy within courses, (c) content coverage, (d) mathematical rigor, and (e) interactions between PSETs' beliefs and prior experiences.

Sequencing Content and Pedagogy across Courses

A major question that comes out of these vignettes, especially from Vignette 1, is the sequencing of mathematics content and pedagogy courses for PSETs. The MET II (CBMS, 2012) recommends PSETs have 12 semester credit hours of coursework dedicated to content and pedagogy for teaching mathematics in both elementary and middle school. Both the SPTM (AMTE, 2017) and the MET II suggest PSETs should know both the mathematics content of elementary and middle grades and the progression of those ideas. However, there is no discussion of effective sequences or progressions of ideas for PSETs in their programs. For example, would it be more beneficial for PSETs to be enrolled in a content course and pedagogy course simultaneously rather than sequentially, allowing more collaboration and discussion across departments for a more coherent and balanced learning progression? We lack guidance on how to balance the right amount of content and pedagogy in an efficient order to optimize PSETs' learning outcomes.

While much research has been done regarding sequencing and learning progressions among K–12 students, little to no analogous guidance is provided for PSETs' preparation at the university level. From discussions between the authors who represent five different universities and from reviewing relevant literature, what is clear is that there is more variation than consistency in the sequencing of courses and intended learning progressions in teacher education programs (Greenberg & Walsh, 2008; Matthews & Seaman, 2007). What is needed is the development and investigation of hypothetical learning trajectories (Simon, 1995) for pedagogy and content for prospective mathematics teacher education.

Integrating Content and Pedagogy within Courses

While the last section considered the sequencing of content and pedagogy across PSETs' courses, another issue that arises from Vignette 1 is the integration of content and pedagogy within courses for PSETs. Often, the ratio of content and pedagogy of the course is directly affected by the department in which the course is housed. According to a survey of elementary teacher preparation programs by Masingila et al. (2012), "the vast majority of the responding schools (78.4%) offer mathematics content courses specifically for prospective elementary teachers, and 88.3% of the schools offering these courses do so through a mathematics department" (p. 351–352). There are benefits and constraints to having PSETs' courses focused on mathematics content housed in mathematics departments. Faculty members in mathematics departments typically have a very deep understanding of the content that allows them to create opportunities for PSETs to learn how to push students to think deeply about mathematics. However, many faculty members in mathematics departments have little to no prior elementary teaching experience, often coming from secondary or undergraduate teaching backgrounds if they had any prior teaching experience themselves (Greenberg & Walsh, 2008; Masingila et al., 2012). Thus, they may not have much expertise in the pedagogy appropriate for teaching mathematics at the elementary level. As is well stated in the MET II (CBMS, 2012):

> A major advance in teacher education is the realization that teachers should study the mathematics they teach in depth, and from the perspective of a teacher. There is widespread agreement among mathematics education researchers and mathematicians that it is not enough for teachers to rely on their past experiences as learners of mathematics. It is also not enough for teachers just to study mathematics that is more advanced than the mathematics they will teach. (p. 23)

Crucially, they go on to recommend that teacher preparation programs should design courses that blend content and pedagogy when possible. In fact, research already shows that positive change is possible in prospective teachers' beliefs and practice after taking an integrated math content and pedagogy course sequence (Hart, 2002). Similarly, AMTE (2017) states,

> Studying mathematics content is necessary but not sufficient. High-quality early childhood teacher preparation programs weave together the learning of mathematics content, the study of specific mathematics pedagogies and effective mathematics instruction, and, at the core, developmental knowledge of children's mathematical thinking and reasoning. (p. 68)

Similar to sequencing, there are clear recommendations on what the overall experience for PSETs should be. However, there is little guidance on how to enact such an experience, essentially leaving critical decisions to the discretion of the program stakeholders. For example, should we start by positioning PSETs themselves as learners of mathematics in a content-specific course, then associating pedagogy with mathematical content through the lens of a teacher in a pedagogy course? Or would it be better to infuse pedagogy into the content from the beginning, developing the content in the manner and order it will be learned by students? We agree that it is important for program stakeholders to have discretion over their programs as they will know their context the best; however, there needs to be more reporting and sharing of what is being done and how effective it is for preparing PSETs to provide their students with positive and meaningful experiences with mathematics.

While these are unanswered questions that require empirical investigation in the future, faculty who work with PSETs may find difficulty implementing the results of that research based on the department of their appointment. For example, Dr. Mathews and Dr. Eduardo both work with PSETs; however, Dr. Eduardo's education department controls the PSET program's course structure, while Dr. Mathews's math department controls the content of courses taught in the math department. Collaboration between the two departments will be key to realizing the benefits of the proposed research.

Content Coverage

In the past, K–12 mathematics in the United States was criticized for being "a mile-wide, inch-deep curriculum," attempting to expose students to far too many topics in a limited amount of time and in very little depth (Common Core State Standards Initiative, n.d.). The introduction of the CCSS-M in 2009 was an attempt to remedy this problem. However, many content courses for PSETs at U.S. universities could be similarly described. As suggested by the NCTM Council for the Accreditation of Educator Preparation standards, "All elementary mathematics specialists should be prepared with depth and breadth in the following mathematical content domains: Number and Operations, Algebra, Geometry and Measurement, Statistics and Probability" (NCTM, 2012, p. 1).

In order to prepare future teachers to teach the myriad topics they will be expected to cover, across multiple content areas and multiple grade levels, many content courses focus on introducing PSETs to as much of this content as possible, as in Dr. Mathews's institution in Vignette 2. Because most colleges and universities have no more than two content courses in which to present this material (Masingila et al., 2012), the pace of the course necessarily moves quickly, leaving little time for investigation or in-depth discussion. A report by the CBMS (2012) advocates this position: "Before beginning to teach, an elementary teacher should study in depth, and from a teacher's perspective, the vast majority of K–5 mathematics, its connections to prekindergarten mathematics, and its connections to grades 6–8 mathematics" (p. 23). The report implies that further investigation into mathematical concepts can be done during in-service professional development, where it can be grounded in the work of teaching. One example is Hill and Ball's (2004) evaluation of in-service teachers' development in their knowledge for teaching mathematics through participating in California's Mathematics Professional Development Institutes. Their analysis suggests that both the length and the focus of the workshop (e.g., mathematical analysis, reasoning, and communication) predicted greater performance gains.

However, other teacher preparation programs make a different decision, choosing instead to focus on a limited number of topics (while skipping others) in favor of spending more time on each topic, like Dr. Eduardo in Vignette 2. This allows for greater investigation and discussion, leading to deeper coverage, but also necessarily means that PSETs will graduate and enter their teaching careers not having been taught everything they will be expected to teach to their students. The SPTM advocates for this choice:

> Even well-prepared beginners [teachers] do not learn all of the mathematics content they need to teach all elementary grade levels in their preservice teacher education programs. However, they must study some key areas, and they should have opportunities to study some of these in depth. Key areas for upper elementary candidates include base-ten numbers, multiplicative structures, fractions and decimals, algebraic thinking, measurement, and geometry. (AMTE, 2017, p. 76)

This choice would also parallel the shift taking place in elementary curricula since the inception of the CCSS-M.

Mathematical Rigor

MTEs teaching elementary content courses may also experience discord in defining "rigor" for the content they teach as illustrated in Vignette 2. While the CCSS-M define rigor as a balance of "conceptual understanding, procedural skills and fluency, and application" (Common Core State Standards Initiative, n.d.), past president of the NCTM, Linda Gojak, describes a lack of consensus even amongst elementary math coaches on the meaning of rigor and its translation into classroom practice (Gojak, 2013). For MTEs employed in mathematics departments, this schism may widen even further.

One perspective on the goal of mathematical instruction is "to help students develop ways of understanding and ways of thinking that are compatible with those that are currently accepted by the mathematics community at large" (Harel, 2008, p. 9). For many mathematicians, rigor is inextricably linked to formal justification and proof due to their prominent status in mathematics and mathematics education (Hanna, 2007). However, there has been a long-standing discussion among mathematicians about the pros and cons of rigorous reasoning and non-rigorous reasoning in proof, in terms of mathematical understanding and problem solving (Kitcher, 1984; Lakatos, 1976; Wittmann, 2020). Currently, the field lacks the research needed to make these difficult, but critical, decisions regarding the meaning and level of mathematical rigor in content courses for PSETs.

Interactions Between PSETs' Beliefs and Experiences and Previously Identified Issues

As CBMS (2012) stated, "Prospective elementary school teachers frequently come to their teacher preparation programs with their own views about what it means to know and do mathematics and how it is learned" (p. 34). In these views are imbued characteristics of the learning experiences they had in the past (Howson et al., 1981; Peace et al., 2018). Further, PSETs bring varying perceptions of self and affective elements, such as identity, beliefs, self-efficacy, emotions, and values that impact their motivation and learning experiences.

The varying views and perceptions PSETs bring into the classroom, as illustrated in Vignette 3 not only impact their motivation and learning experiences, but also interplay with the

MTE's pedagogical decisions regarding sequencing, scope, rigor, and the MTE's beliefs and values. Dr. Tusi noticed that the varying views and perceptions of selves PSETs brought into his classroom interacted with his own beliefs, such as valuing conceptual understanding and students' mathematical thinking. Moreover, he felt tension between wanting to spend time to explicitly address these varying views and perceptions and having to cover content in a limited time frame.

Some studies have shown that PSETs often describe themselves as not having been successful in school mathematics in the past or disliking mathematics, which is usually correlated with their math anxiety level (e.g., Hembree, 1990). In the AMTE's (2017) SPTM, developing students' positive dispositions towards mathematics is emphasized; yet, how to accomplish this is not documented. Although many policy documents emphasize the importance of these issues, not many provide guidance for how to address these in PSET content courses. Also, extant literature on teachers' dispositions and perceptions of self, such as identity, beliefs, and emotions, were mainly studied in the context of a pedagogy course for PSETs or professional development of in-service teachers (e.g., Gomez, 2018; Hodgen & Askew, 2007). Future research addressing PSETs' views and perceptions of self in the context of content courses and ways in which MTEs can support PSETs' positive learning experiences are needed. Further, studies on MTEs' views of learning, doing, and teaching mathematics and their interactions with those of PSETs can inform MTEs, such as Dr. Tusi, in their teaching practice.

Discussion

This piece has sought to highlight areas that could benefit from more discussion and research with the goal of creating possible models of PSET education with respect to mathematics content and pedagogy. Based on the issues discussed in the vignettes and experienced by the authors, we would like to make the following suggestions for future research on the preparation of future elementary mathematics teachers:

- research on the benefits and drawbacks of the various formats of programs for preparing PSETs
- research examining the differential effects of varying amounts and sequencing of content coverage versus pedagogy coverage both within and across courses on PSETs' knowledge, practice, and beliefs
- research examining the differential effects of varying levels of mathematical rigor on PSET's knowledge, practice, and beliefs
- research on effective ways in which MTEs could help PSETs develop positive dispositions towards self and mathematics

We would also like to mention that these issues also merit discussion in relation to the preparation of MTEs. We hope that this discussion will help to spark further work in considerations of program design to prepare elementary teachers to teach mathematics.

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