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Review of *Mathematical Understanding for Secondary Teaching: A Framework and Classroom-Based Situations* by Heid, Wilson, and Blume (2015)

Bridget Druken

Imagine a high school mathematics teacher looking for big mathematical ideas while planning their secondary mathematics lessons. They might come across questions like the following and wonder how they might best respond to support their student's learning: Why is it that when you multiply two negative numbers together, you get a positive number answer? How can we think about measures of center using box plots? Why is a number raised to the zero power 1 and not 0, and, what is 0 raised to the 0 power? *This review of* Mathematical Understanding for Secondary Teaching: A Framework and Classroom-Based Situations *by Heid, Wilson, and Blume (2015) addresses how to go about responding.*

Mathematical Understanding for Secondary Teaching: A Framework and Classroom-Based Situations (MUST) edited by Heid et al. (2015) offered a new way to think about mathematics needed to teach secondary school. Exactly what content is optimal for secondary teachers to know has long been questioned in mathematics teacher education. Heid et al.'s work extended work on the mathematical knowledge for teaching elementary mathematics to include secondary mathematics (Hill et al., 2008). This book is a welcomed addition to anyone involved with secondary mathematics teacher education.

Teaching mathematics involves more than assenting to a list of mathematical knowledge and skills, the authors contend. Promoting students' understanding of, expertise with, and appreciation for mathematics requires a particular kind of understanding.

Bridget Druken *is an associate professor of mathematics education at California State University, Fullerton. Her primary research focuses on using mathematics lesson study as a form of professional development and supporting future teachers of K-12 mathematics in learning mathematics for teaching.*

> A mathematician may prove a theorem, and an architect may perform geometric calculations. For these users of mathematics, it is sufficient that they have the skills and abilities for the task at hand. But a teacher's work includes these tasks as well as interpreting students' mathematics, knowing where students are on the path of mathematical understanding, developing multiple representations of mathematical concepts tailored to students' understandings, using their advanced mathematical understanding to craft tasks and examples with a specific set of characteristics, and so on. (Heid et al., 2015, p. 9)

Consequently, the MUST framework focuses on mathematical understanding rather than mathematical knowledge. In a welcomed perspective, the authors distinguish understanding from knowledge by suggesting that understanding involves knowing mathematics as dynamic rather than static. This intentional attempt reflects mathematical understanding as a growing and evolving effort, one that changes over a person's career.

MUST was designed from visits to secondary mathematics classrooms where questions like those in the opening inevitably arise. Written primarily for mathematics educators interested in supporting future and current secondary teachers in deepening their mathematical understandings, this book is important due to its ability to (re)frame one's understanding of mathematical knowing for secondary teaching. This involves shifting away from solely examining mathematical content (e.g., topics, skills, concepts) to include processes (e.g., practices and habits of thinking like a mathematician). This work also provides a framework to continue identifying and deepening new mathematical understandings for secondary teaching rather than providing an exhaustive list of skills and knowledge.

Many individuals contributed to developing the framework, including practicing mathematics teachers and leaders, mathematics teacher educators and researchers, mathematicians, and preservice teachers. Three national conferences were held to provide opportunities for feedback. The inclusion of different stakeholders who support secondary mathematics teachers is appreciated.

Clear organization is evident from the start: Part 1 provides information on the MUST framework, and Part 2 contains over 40 situations of their MUST framework in action, which are loosely grouped by algebraic reasoning, geometrical reasoning, statistical reasoning, calculus, and mathematical induction.

Part 1

Part 1 presents the MUST framework in seven readable chapters. Rooted in practice, the authors developed the MUST framework by observing secondary mathematics classrooms and noting questions or events that arose while teaching. This effort to ground the framework development in actual events that occurred in secondary mathematics classrooms is appreciated. Importantly, this framework does not focus on determining curriculum, pedagogies, or standard mathematical concepts. Instead, the framework helps the reader analyze mathematics situations deeply, "without getting into the specifics of what a teacher and his or her students might do with that mathematics" (Heid et al., 2015, p. 2).

Three perspectives comprise the MUST framework, as explained in Chapter 2:

- *mathematical proficiency,* which builds on the five strands of mathematical proficiencies described by the National Research Council (2001) in *Adding It Up: Helping Children Learn Mathematics*
- *mathematical activity*, including mathematical noticing, mathematical reasoning, and mathematical creating, builds on the *Principles and Standards for School Mathematics* from the National Council of Teachers of Mathematics (NCTM; 2000) and the *Standards for Mathematical Practice* (Common Core State Standards Initiative, 2010)
- *mathematical context*, which refers to facilitating students' mathematical learning.

A few examples of mathematical context include knowing and using the curriculum, accessing and understanding the mathematical thinking of learners, and assessing the mathematical knowledge of learners. By attending to these three perspectives, understanding for teaching secondary mathematics can be approached as a developing quality rather than a static entity or endpoint.

Chapter 3 details the components of a situation and their process for the development of a situation. A *situation* is a mathematical description, based on an actual event that occurred in the practice of teaching. Situations provide concrete examples of the MUST framework in action. Chapter 4 describes building the framework from situations, Chapter 5 describes the creation of new situations as inquiry, and Chapter 6 summarizes the many uses for the MUST framework.

Part 2

Part 2 provides 43 situations of the framework in action. Each situation includes:

- a *prompt*, which is a motivating mathematical question that might arise in teaching mathematics (e.g., a question raised by a student, student response to a teacher's question, common error made by a student)
- *commentary* on the particular classroom prompt (and sometimes post-commentary that further summarizes and connects several ideas)
- a *set of mathematical foci,* or multiple ways to address the underlying ideas and topics identified by the prompt

While challenging to bring the complexities of classroom practice to life within a physical book, one of the great parts of the framework is the post-commentary remarks that further describe the particular, original classroom prompt and connects it to other mathematical ideas.

It is important to note that situations do not provide pedagogical advice about what mathematics teachers should discuss or do with students. Rather, situations aim to explicate the mathematical understandings useful for the teacher. The authors state that, “We describe the mathematics itself and leave it to the teacher or mathematics educator to decide what to use and how to do so” (Heid et al., 2015, p. 3). This intentional decision emphasizes mathematical foci. Additionally, the authors wanted to honor that students and school settings determine what a teacher does with mathematical understanding(s).

The reader will find many mathematical situations that can be read in almost any order. Sample algebraic-based situations include: properties of complex numbers, zero-product property, graphing inequalities, and translation of functions. Sample geometric-based situations include: area of plane figures, area of sectors of a circle, similarity, Pythagorean theorem, circumscribing polygons, and trigonometric identities. Situations about statistical reasoning include: mean and median, representing standard deviation, and sample variance and population variance. A few remaining situations include the product rule for differentiation and proof by mathematical induction. While different in their focus on understanding, situations are united in their prompt–commentary–mathematical foci–post-commentary structure, making it easy for the reader to digest.

If the reader is interested in the development of the framework, the first six chapters in Part 1 will be useful. If the reader is interested in examples of the framework in action, the 40+ chapters in Part 2 will be useful.

A Specific MUST Example - Solving Quadratic Equations

Take the situation titled “Solving Quadratic Equations” by Jeanne Shimizu, Sarah Donaldson, Kelly Edenfield, and Erik Jacobson (Heid et al., 2015). This situation, initiated by a relatable algebraic occurrence seen in the classroom, weaves together several foci that are key to understanding the initial prompt. The prompt beginning this seven-page chapter is the following:

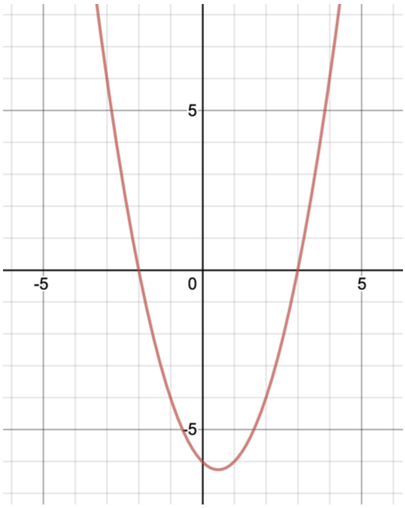
In an Algebra 1 class some students began solving a quadratic equation as follows:

Solve for *x*:

$$x^2 = x + 6$$
$$\sqrt{x^2} = \sqrt{x + 6}$$
$$x = \sqrt{x + 6}$$

They stopped at this point, not knowing what to do next. (Heid et al., 2015, p. 249)

Figure 1
The graph of $y = x^2 - x - 6$.



Next, the commentary provides an overview of the mathematical foci that follow, an appreciated organizer for the reader. The five mathematical foci come next. While not meant to be exhaustive, the foci elaborate on key ideas related to the prompt. Mathematical Focus 1 states: "Factoring and using the zero-product property can be used to solve many quadratic equations" (p. 250). What follows is a discussion on the zero-product property as a consequence of axioms of our real number system. Additional foci include:

- Mathematical Focus 2: “All quadratic equations can be solved by completing the square or by using the quadratic formula” (Heid et al., 2015, p. 251).
- Mathematical Focus 3: “A geometric analogy to an area model can be used to represent quadratic equations and their solutions” (Heid et al., 2015, p. 252).
- Mathematical Focus 4: “Solving an equation using algebraic manipulation requires that equivalence is maintained between each form of the equation” (Heid et al., 2015, p. 252).
- Mathematical Focus 5: “Approximate solutions of equations can be found by graphically determining the zeros of the associated function” (Heid et al., 2015, p. 254).

These mathematical foci dig into big ideas about extraneous solutions and maintaining equivalence by applying an invertible function to both sides of an equation. The foci also make connections from solutions of an equation to zeros of a function of x, where the x-intercepts of a function are the x-value(s) for which the function $f(x)$ evaluates to zero. The authors note that one way to solve the Prompt is to treat each side of the equation as two functions in and of themselves, namely that $f(x) = x^2$ and $g(x) = x + 6$, and find the points of intersection of the graphed functions.

Together, the prompt, set of foci, and commentaries demonstrate ways to link static mathematical content to dynamic understanding through a focus on proficiencies, activities, and contexts.

Users of This Book

The MUST framework is useful for several audiences. In refining their framework, the authors held a conference at the University of Georgia, which invited over 60 participants who proposed ways of using the framework. Chapter 6 provides creative suggestions on ways to use the framework:

- research in mathematics education

- professional development—preservice teacher preparation
- professional development—in-service teacher learning
- teaching mathematics content courses
- graduate mathematics education
- assessment

Use in Teaching Mathematics Undergraduate Content Courses

Instructors preparing preservice or in-service secondary math teachers might find this work useful for planning learning experiences that connect important mathematical ideas, both for individual lessons and overall course organization. Undergraduate students might find these situations useful because they connect mathematical ideas across different mathematics content courses. Additionally, undergraduates who do not have access to a high school mathematics classroom will appreciate the authors' move to set the framework in secondary mathematics classrooms. Due to the theoretical framework, each situation provides the opportunity to think about the mathematical proficiency, mathematical activity, and mathematical context of teaching.

For example, an instructor might present students a prompt followed by individual or group exploration and discussions. After students have the opportunity to consider mathematics relevant to the prompt, further discussion of the mathematical foci could occur whole-class. Engagement could unfold in a face-to-face class or in an asynchronous online discussion-based learning environment. Homework might involve going into greater depth on a particular foci, assigning other situations to read and summarize, or presenting a prompt and asking other students to anticipate potential foci.

Use in Teaching Mathematics Graduate Content Courses

Graduate students would benefit from deepening their mathematical reasoning using MUST while taking advanced mathematics courses. Instructors could assign different situations to graduate students and invite them to study and discuss the situation, which would allow them to learn about mathematical understandings needed for teaching. It is also powerful for graduate students who may teach at the community college level to consider the mathematical proficiencies, activities, and contexts simultaneously. They could also create new situations to further their understanding of teaching secondary mathematics, particularly in areas with limited situations, such as introduction to proof courses, calculus, statistics, and probability. These contributions could then be shared broadly through publications.

Use in Professional Development

Those involved in teacher professional development could use situations to practice noticing salient aspects of a mathematical activity, orchestrate discussions on underlying mathematical foci, and rehearse how to respond to their students (Grossman et al., 2009). Teachers could extend current situations by creating educational technologies or writing an historical background to accompany each situation—what technologies (e.g., Desmos, GeoGebra, or CODAP) might help connect mathematical foci within a situation? What motivated the field to develop this mathematical idea? How might this situation be applied to applied mathematics and statistics? Following the authors' guidance in chapter 5, "Creating New Situations as Inquiry," teachers could design new situations that are connected to their schools' communities and learning goals, perhaps engaging in a lesson study to guide their inquiry (Lewis et al., 2009).

Challenges

The authors noted several limitations of the framework that include observing limited classroom settings, using technology sparingly across situations, using mainly algebra classrooms to generate each situation, and observing few classes that were using reform curricula. Additionally, historical stories, cultural contexts, and connections to the real world that shape these understandings are underdeveloped. While mathematical contexts played a role in their framework, a more prominent connection to broader communities outside of the classroom could enrich each situation along with discussions on relevant technologies useful for understanding. Possible expansions of this flexible framework may attempt to answer questions like: How might the framework be extended to include connections to histories, cultures, and real-world scenarios? and What technologies could be used in conjunction with each prompt to facilitate development of each situation?

Perhaps one of the most challenging things to understand about this book is its attempt to develop a framework about mathematical understanding for secondary teaching rather than identify specific knowledge about mathematical understanding for secondary teaching. This is due to the authors' belief that "Mathematical understanding for teaching should grow and deepen over the course of a teacher's career, and the lenses that comprise our framework characterize the nature of the mathematical proficiencies, actions, and contexts that set those understandings apart" (Heid et al., p. 5). Considering this challenge, the book contributed strongly to the mathematics education field.

Closing

This 50-chapter book is a laudable effort toward addressing the challenge of understanding what a secondary mathematics teacher should know for teaching mathematics. The MUST framework contributes well to broadening the concept of mathematical knowledge from a list of mathematical skills and knowledge to include mathematical proficiencies, activities, and contexts. Overall, this book is appropriate for those interested in connecting advanced mathematics to high school mathematics.

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