Appendix

Interactivity Flow Chart for Triadic Communication

Line #	*C*	*C* ↔ *B*	*B*	*B* ↔ *J*	*J*	*J* ↔ *C*
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Discoursing Mathematically: Using Discourse Analysis to Develop a Sociocritical Perspective of Mathematics Education

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This article explores how the concepts of discourse and its methodological extension discourse analysis can help mathematics educators re-conceptualize their practices using a sociocultural view of learning. It provides conceptual and methodological tools as well as activities that can be helpful in mathematics methods courses and professional development sessions aimed at developing a more situated and social view of mathematical discourse and its relationship to student learning, particularly how mathematical discourse relates to Language Minority Students (LMS). In this article, I discuss the main features of discourse as a framework for mathematics educators and how participants in a cross-site research center collectively engaged and developed a more robust understanding of the significance of discourse and discourse analysis for understanding mathematics as a sociocultural practice. This article describes learning activities whose instructional goal is to develop a sociocritical understanding of language and mathematics. The activities presented here can be adopted as a model for engaging mathematics teacher educators and mathematics teachers to deepen their understanding of the inextricable link between language and mathematics, and of mathematics as a cultural and political activity.

The preparation of teachers for linguistically and culturally diverse populations has been the subject of a growing body of research and discussion over the last two decades (Brisk, 2008; Cochran-Smith, Fieman-Nemser, McIntyre, & Demers, 2008). The relatively recent emphasis on this issue in the research community has taken place primarily because of the rapidly changing

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demographics in the nation's student population accompanied by the persistent disparities that exist in educational achievement, resources, and life opportunities between Language Minority Students (LMS) and their majority counterparts. According to a report from the National Center of Educational Statistics (NCES, 2010), in 2008, 21% of all children aged 5 to 17 spoke a language other than English at home. As it stands now, there are an insufficient number of teachers who are adequately and appropriately prepared with the skills and knowledge to teach LMS (Lucas & Grinberg, 2008). Since achievement in mathematics is highly dependent on teachers' capabilities, the under-preparedness of teachers does not bode well for LMS who are not receiving the support they need to perform well in mathematics (Gutiérrez, 2002).

Although research has pointed to the importance of linguistically responsive learning environments for LMS in mathematics (e.g., Khisty, 2002; Moschovich, 1999a) and to practices teachers can use to facilitate LMS learning of mathematics (e.g., Moschovich, 1999b), there still remains a question of how to prepare and support teachers in creating such learning environments. In fact, almost no research has been conducted on the preparation of teachers to teach LMS (Lucas & Grinberg, 2008; Zeichner, 2005). This is particularly true in the domain of mathematics, as most research on mathematics teacher preparation has focused on preservice teachers' knowledge and beliefs about mathematics, their applications of constructivist principles, and understanding of problem-solving processes and skills (Lester, 2007; Llinares & Krainer, 2006). Discussions in mathematics education have not given sufficient attention to developing teacher knowledge related to teaching LMS, and most mathematics teacher educators do not have the background knowledge necessary to prepare teachers to teach mathematics to LMS. As a result, preservice teachers enter the profession having little knowledge about the needs, resources, and supports required to effectively teach mathematics to LMS (Chval & Pinnow, 2010).

Teachers must have a deep knowledge of the linguistic and cultural demands that are unique to the teaching and learning of mathematics. This becomes more important when students speak (or are learning) more than one language (Valdés, Bunch, Snow, Lee, & Matos, 2005). Although the importance of language and

mathematical discourse[1] in the process of teaching and learning mathematics has gained considerable attention in recent years both in monolingual (e.g., Cobb, Yackel, & McClain, 2000) and bi/multilingual (e.g., Moschovich, 2007; Setati, 2005) contexts, it has not been given sufficient attention in teacher preparation programs. Substantial language and discourse content is absent in most mathematics teaching courses for preservice teachers because language is typically treated as a subject in teacher education and is separated from the content subjects. In addition, mathematics teacher educators need professional development in order to include language and discourse issues in their teacher preparation courses.

In this article, I provide conceptual and methodological tools as well as activities that can be helpful in mathematics methods courses and professional development sessions aimed at developing a more situated and social view of mathematical discourse and its relationship to student learning, particularly how mathematical discourse relates to LMS. I explore how the concepts of *discourse* and its methodological extension *discourse analysis* can help mathematics educators re-conceptualize their disciplinary field and student learning.

First, I provide the context in which these methodological and conceptual tools were developed. Next, I outline some of the main features of *discourse* as a framework for mathematics educators and teachers. Drawing on Gee's definitions of primary and secondary discourses (Gee, 1996) as well as the *material, activity, semiotic, and sociocultural* (MASS) dimensions of discourse analysis and learning (Gee & Green, 1998), I show how the concepts of discourse and discourse analysis are particularly relevant in mathematics education and, more specifically, mathematics teacher preparation. I conclude with implications for mathematics teacher preparation and directions for future research.

Context

In 2004, as part of NSF's Centers for Learning and Teaching (CLT) initiative, the Center for Mathematics Education of Latinas/os (CEMELA) received a five-year grant to train doctoral students across four campuses who would focus on the intersections of mathematics, language, and culture especially in the context of bilingual, Latina/o children in urban settings. At two

of the sites, CEMELA conducted after-school mathematics clubs to develop mathematics literacy based on the learning principles described later in this article and community expertise. As part of their training/socialization, doctoral students, faculty across disciplines (mathematics, mathematics education, literacy), and practitioners participated in summer intensives dedicated to the topics of mathematics and discourse (total of 65 participants). It was the central subject of a six day intensive "school" held at the University of Illinois at Chicago in the summer of 2007. Over 97% of the participants reported that the school "helped develop skills to analyze discourse processes" (LeCroy & Milligan, 2007, p. 28) especially as they relate to the mathematics education of bilingual students. One participant commented, "I learned about discourse analysis and aspects of bilingualism that apply in the classroom" (p. 29) According to other students, the activities were "well-developed to learn difficult concepts such as Gee's discourse" (p. 32), they were "useful" (p. 33), and "more" (p. 33) activities like this should be done. More specifically, the Baseball Language Learner (BLL) activity, which I will discuss in more detail later, was discussed as the most effective for making the distinction between *language* and *discourse* clear, "most helpful were the [Baseball Language Learner] activity combined with the ideas of diverse communities that consider language and cultural context." In this article, I will provide a detailed account of how the participants and I engaged in discussions of discourse, discourse analysis, and mathematics through the BLL activity.

I and other doctoral fellows, who are now in faculty positions, have continued to use these learning activities in a variety of teacher education and bilingual/ESL courses for the purposes of developing teacher awareness about the relationship of language and mathematics. In the following sections, I discuss the four fundamental tenets of discourse and discourse analysis that drive this professional development and illustrate how the issues were discussed at the summer intensive.

From Language to Discourse: Four Fundamental Questions

Many teachers, including doctoral students, came to the discussion of language and mathematics with "folk theories" of what counts as language. When asked to define "language" there was unanimous agreement in that language is either the spoken or

written word for the purpose of communication. In the following section, I show the activities and process that the participants undertook in order to reframe this intuitively yet deceptively "correct" view of language and how they progressively moved towards less intuitive yet more profound and critical notions of language as "discourse." I show how the participants and I moved from "what people say" to critical issues of "values, beliefs, and power relations." Given that my research questions and projects are situated in Latina/o urban settings with large populations of LMS, the importance of teacher beliefs about the nature and function of language in relation to mathematics has significant implications for student learning, instruction, and ultimately outcomes (Razfar, 2003).

1) What do people say?

In examining the salience of discourse and discourse analysis for the mathematics education communities, it is important to consider some of the fundamental principles and questions that guide discourse analysts as they look at transcripts of talk irrespective of their field or discipline. Of central concern to practitioners and researchers is that discourse analysis is one of the most important tools for organizing and assessing learning and development especially from a cultural historical perspective. The first and perhaps most obvious question is, *what do people say?* Linguists have traditionally referred to this as the *code* or the more formal and explicit features of language, namely the structure. While for linguists these utterances do not typically take place in naturalistic situations, the idea that this is the most descriptive aspect of language form applies, and all discourse analysis necessarily accounts for this dimension. More specifically, this refers to the most apparent features of language such as sounds, pronunciation (phonetic and phonological aspects), words (lexical choice), morphology, and grammar (syntax). If this dimension were extended to typical interactions, this would include the spoken utterances attributed to each speaker and the obvious turns that speakers take within an episode of talk. In order to make this point I provided a transcript of talk to all participants. The first snippet of discourse that is presented is strictly transcribed based on spoken words (code) and all performative aspects are missing. It is an interaction between Juan (denoted J in the transcript), one

of the kids in one of the after-school clubs, a graduate assistant (denoted G in the transcript), and a mechanic (denoted M in the transcript) about the hydraulics of a car:

1 G: So if you wanted to make a car a low-rider?
2 [0.5 second pause] Like make it so that it is
3 lower.

4 M: On a regular car you would actually have to
5 do a lot of suspension work. One of the first
6 things that you want to do- there are different
7 things that you want to do. You can start with
8 airbags where you compress the air. You
9 know, and then they're actually bags itself
10 where you just compress the air, it deflates
11 'em and increases the air and that'll make the
12 car go up and down. The other one hydraulics
13 and that's actually based on fluid. Fluid is
14 actually what's going to go through there. It's
15 going to actually put pressure on the cylinder.
16 Once the fluid puts pressure on the cylinder,
17 the cylinder will go up. [inaudible] makes the
18 cylinder go down. So basically you have
19 those two. Do you want to go with airbags or
20 do you want to go with hydraulics?

While Juan is present in the interaction, he is not visible in the transcript. After reviewing this clip, and discussing it in small groups, participants drew conclusions based on the code available in the transcript. When this brief exchange was analyzed, participants concluded that there were only two speakers: One was asking a question, and the other was responding. One speaker is or *appears* to be clarifying the initial question (line 1) where the concept of "low rider" is extended, "like make it so that it is lower" (lines 2–3). Structurally, everybody agreed that the words being used were English and followed normative rules of English morphology and syntax. Some even used the transcript to identify various parts of speech (nouns, verbs, prepositions, etc.), word order, subject/object functions, modals, and even the logical connectors. Participants arguably used the more common/folk approach to what counts as language and drew typical and

uncontroversial conclusions from the code. The following sections illustrate why this approach is not sufficient and how the transcription exercise made this visible to participants.

How do people say what they say?

If the analysis were to stop here, it would clearly be insufficient in terms of the second and third questions that are central to discourse analysis which are: *How do people say what they say? And what do they mean?* The second question has historically been the domain of applied linguists and sociolinguists and is traditionally referred to as *performance*. In general, this is actual language use in real communicative situations and is concerned with how speakers draw on contextual cues to communicate. In addition, performance also consists of *prosodic* dimensions of language use like tone, intonation, loudness, pitch, and rhythm. This can also include gestures, facial expressions, and other non-verbal acts which make transcription quite challenging and impossible without video. Prosody offers an initial glimpse into the affective stances speakers assume within discourse frames. Participants were then asked to reflect on a different transcription of the same speech event that took into account the *performative* qualities. Lines (1-3) from the previous transcript are "re-presented" below (G=Graduate Assistant; J=Juan):[2]

G: So if you wanted to make a ca:::r (.5 sec) a (.5 sec) a low rider (rapid voice, falling intonation), li:ke (.5 sec) ma:ke it so that it is lower.

J: [Juan nodding] [yeah]

After reflection and discussion, several issues became clear. First, what initially looked like a question followed by a clarification for the mechanic appears to be some type of scaffolding directed at Juan, a student in the after-school club. In comparing the first transcript with the second, everybody noticed the invisibility of Juan in the first transcript, which was strictly code. As the discussion moved from an analysis of code to an analysis of performance, Juan's role in the interaction became more apparent. One participant made the following observation, "in the first transcript there were only two speakers, but in the second there are three…we couldn't see the non-verbal." Several talked about the importance of video, but even video can be

limited as I discuss in the next section on meaning. The overlapping talk whereby the graduate student assumes the floor interspersed with non-verbal acknowledgements from Juan is critical to the analysis. Furthermore, there is clear hedging (deliberate pause followed by a rapid voice and falling intonation) surrounding the word "low rider." As the participants moved in this direction, there were more questions about the meaning and functions of the words described in the initial phase of the analysis. The main question that was raised was, "If Juan had already acknowledged the use of the term low-rider and from previous turns and interactions all participants use the term freely, what is the purpose of the 'clarification'?"

3) What do people mean?

This question led us to the central and arguably most contested interpretive question for discourse analysts and that is, *what do people mean?* If one assumes that meaning is fixed, absolute, and independent from the situation in which it occurs, then there is little argument; however, meaning is situated and necessarily dependent on the *footing* of the participants within a particular *frame* (Goffman, 1981).[3] The question that arises: Does the graduate assistant in the interaction, using the term "low rider," share the same footing with the other participants? In addition, participants invoke intentions and purposes that are often hidden from the immediate and apparent discourse. It is essential for us to historically locate the term "low rider" as used by the immediate participants and well beyond, in order to grapple with issues of purpose and intention. Speakers often draw on multiple signs and symbols in multiple modalities available to them in order to achieve higher degrees of shared meaning or what Bakhtin called *intersubjectivity* (Holquist, 1990).[4]

From the above example, one might argue that the hesitation surrounding the word "low rider" is not about referential meaning or shared understanding, but more about speech rights and identities indexed by the use of the term. Does the speaker feel a right to freely use the term "low rider"? Does the speaker have a discourse affinity with the term? One participant noted, "I don't think she is comfortable using the term [low rider]…maybe she is nervous." The issue of *speech rights* has serious implications for discourse and identity. It impacts the what, who, and how of

allowable discourse. In this case, the graduate student is a White female, who although fluent in Spanish and having lived in a Latin American country for a long period of time, appeared to be hesitant and aware that she could be encroaching upon implicit cultural boundaries. This conversation proved to be the most unsettling in terms of participants' assumptions about language, discourse, and identity; nevertheless, it made issues of meaning, intention, and identity more visible. One participant commented, "Discourse is more than just words, it is who we are and who we get to be." Thus, meaning-making is necessarily embedded within the values, beliefs, and historical relations of power; an aspect of discourse that Gee has often referred to as *Discourse* (Gee, 1996). This dimension is often beyond the apparent text and requires deeper ethnographic relations between the researcher and participating community members in order to conduct more authentic analysis of meaning-making. This leads to the final premise of what constitutes discourse.

4) How do values, beliefs, social, institutional relations of power mediate meaning?

This question constitutes *the critical* dimension, and its importance with respect to discourse analysis cannot be underscored enough especially *vis a vis* mathematical discourses. It is the central question when it comes to understanding how some practices are more valued, privileged, and attributed greater legitimacy than others. This is particularly salient when dealing with non-dominant dialects, languages, and cultures that are prevalent in urban settings. Issues of racial, economic, and gender inequity and access are no longer variables that can be placed on the periphery of analysis, but rather take on a central role. Identities and ideologies become fore-grounded in the analysis of talk and text. Street and Baker (2005) call this the ideological model of numeracy which is an extension of Street's ideological approach to literacy. In the context of the questions posed by researchers and others looking at mathematical and scientific practices in non-classroom settings, it is particularly salient when one considers what gets counted as legitimate mathematics.

The process of interpreting the meaning-making of people is continuous, subject to constant revision, and dependent on how much of an ethnographic perspective the analysis presumes. A

teacher as an ethnographer (Gonzalez, Moll, & Amanti, 2005) is a powerful metaphor that brings together the aims of discourse analysis and the practitioner in the classroom. Given the emphasis on meaning-making, mathematical practices are also viewed in this light. In the remainder of this article, I will explore how discourse analysis can be a valuable tool for understanding mathematical practices as situated problem solving that largely depend on local cultural contexts and symbol systems.

Learning as Shifts in Discursive Identities: Primary versus Secondary Discourses

At this point in the discussion within the professional development, an argument in favor of "discourse" versus narrow conceptions of "language" had emerged. In external evaluations conducted after the session, nearly all participants "strongly agreed" that the transcript exercise was an effective tool for this purpose. When participants considered the four dimensions/questions of discourse analysis raised above, it became evident that the notion of discourse (as opposed to "language") afforded a more holistic view of human meaning-making. Yet, the connection to learning, teaching, and instruction is not self-evident. One participant commented, "So we analyze all of this discourse, but how does it help a teacher in the classroom…and where's the math?" Discourse analysts have long argued that learning itself is best understood as shifts in discourse over time, especially the appropriation of *discursive identities* (Brown, 2004; Rogoff, 2003; Wortham, 2003). The critical point here is "over time" and according to Brown, Reveles, and Kelly (2005), "research in education needs to examine identity development, learning, and affiliation across multiple timescales." (p. 783). Understanding how discursive identities change over time is difficult for participants to appreciate in a short course or professional development session (however intensive). Doctoral fellows and practitioners, however, were able to develop such a perspective over the course of four years of ethnographic work in the after-school clubs.

As practitioners and researchers embrace the notion of learning as shifts in discursive identities, a couple of questions remain: What kinds of discourse constitute mathematics? More generally, where do formalized discourses (i.e., those that are

learned in schools) fit in relation to everyday discourses? Although human beings undergo a life-long process of language socialization, not all discourses are equivalent both in terms of the process and purpose of appropriation. Discourses that seem more natural or are appropriated as a result of spontaneous interaction are distinct from those that are appropriated through participation in formalized institutional settings. For example, the learning of one's native, national language (e.g., Spanish, English, etc.) is different from learning biological nomenclatures or geometric theorems.

With regards to this distinction there is a clear delineation between *primary discourses* and *secondary discourses* (Gee, 1996). In the fields of cognition and second language acquisition (SLA), one of the most contentious arguments has been the distinction between *learning* and *acquisition* (Krashen, 2003; White, 1987). Learning is generally conscious, formal, and explicit, while acquisition is subconscious, informal, and implicit. In contrast to most cognitivists and SLA perspectives who locate both processes within the individual, Gee takes a more situated and sociocultural view on the issue; he argues that acquisition, or primary discourse, is good for performance, and learning is good for meta-level knowledge (secondary discourse). This distinction is important as one considers the features of what constitutes mathematical discourse in relation to learning in informal and formal settings. According to Gee (1996), primary discourses "are those to which people are apprenticed early in life during their primary socialization as members of particular families within their socio-cultural setting" (p. 137); and secondary discourses are "those to which people are apprenticed as part of their socialization within various local, state and national groups and institutions outside early and peer group socialisation, for example, churches, schools, etc." (p. 133). Secondary discourses have the properties of a more generalizable cultural model, are more explicitly taught, and are less dependent on the immediate situation for access by a larger audience.

If algebraic discourse is considered as an example of discourse appropriated through school, then the symbol x in $x+2=7$ is understood by algebraic discourse community members as representing the unknown within an equation as opposed to an arbitrary letter. Members of this community may also assume that

in this case x has a single value and they must follow certain rules to find the answer (all school like practices). Furthermore, for those who have appropriated geometric discourses such as the Pythagorean Theorem ($x^2 + y^2 = z^2$), the x and the y represent the two adjacent sides that form the right angle (or *legs*) and the z represents the hypotenuse. Thus, mathematical symbols gain specialized meanings within multiple domains of mathematics. These literacies serve as mediational tools in novel problem-solving situations, and literate discourses tend to be more generalizable problem-solving tools (Sfard, 2002).

These types of "formal" mathematics discourses would qualify as secondary discourses. This does not, however, mean that primary discourses (especially informal numeracy and mathematical practices) are separate and unrelated to the development of secondary discourses (formal and specialized mathematical practices). Given that learning from a sociocultural point of view is historically continuous, all secondary discourses are either formally or informally connected to the learner's primary discourses. However, this does not mean that primary discourses are always optimally leveraged to develop secondary discourses, especially in formal, "school-like," instructional settings. Ideally, secondary discourses would be explicitly developed through primary discourses, which require a greater understanding of learners' primary discursive identities.

Mathematics could be considered a specialized secondary discourse developed by people for specific purposes. It is important to explicitly define the discursive markers of each in order to have such a phenomenon as mathematics or to have a conversation about what counts as mathematics. For example, one possible definition is that mathematics is a special type of discourse that deals with quantities and shapes (i.e., a secondary discourse); however, there are many ways in which this can be done depending on the context as many studies have shown (e.g., Cole, 1996; Lave, 1988; Scribner & Cole, 1981). Although this definition (or any definition) of a domain of knowledge is not without contestation and would undoubtedly be considered a narrow view of what counts as mathematics, it is an example of one way that mathematics discourse distinguishes itself from other forms of talk. I now turn to how the connection between discourse and learning is made more explicit in the context of professional

development.

Connecting Discourse to Learning and Development

In connecting sociocultural views of learning and development (especially CHAT[5]) with the discourse analysis issues discussed above, there are five issues to consider: (a) activity goals, (b) mediational tools (symbolic/visual), (c) the action/object to meaning ratio, (d) situated versus literate discourses, and (e) "transfer" or cross-situational discourses. As far as mediation is concerned, it is well established within Vygotskian and neo-Vygotskian traditions that learning proceeds from the *inter*personal plane toward the *intra*personal plane through the active use of symbolic and visual artifacts. The material and ideational tools that human beings draw on are historically and socially constituted and become organized as Discourses across generations of actors.

According to Wertsch (1998), all human meaning-making is purposeful, goal driven, and rule governed. These factors are assumed features of discursive practices regardless of the setting. In his work on children in play situations, Vygotsky (1978; 1987) argued that one of the primary measures of development are the shifts in the *action to meaning* ratio. In the early stages of learning, the object(s)/action(s) dominate the child's ability to make meaning. For example, the presence of a cup filled with some type of liquid would prompt a child to say "water" because the set of object(s)/action(s) dominate the use of signs and symbols which are highly context dependent in the early stages of development. However, over time the meaning of the phonetic sounds for the word "water" (/wɔtər/) become less dependent on the presence of object(s)/action(s). Through the mediation of more expert others and the use of symbolic tools, learners develop the ability to regulate meaning without relying on context (see Figure 1):

Figure 1. The shift in Action/Meaning Ratio.

The appropriation of primary and secondary discourses happen in much the same way with one difference: secondary

discourses represent a greater level of abstraction which means the ratio of action to meaning is slanted toward meaning. This gives secondary discourses the added utility of having cross-situational applicability. However, when mathematical and scientific practices (i.e., the disciplinary activities of a community of scholars) are conceptualized as "discourse" or more precisely a secondary discourse, then it follows that one cannot reach more abstract levels without the mediation of objects and actions. A clear implication of this point is how sometimes mathematics learning in formal instructional settings is organized as discrete activities in the form of text-based lessons or reductive worksheets. These types of activities serve to present mathematics practices as a set of isolated skills devoid of culturally situated purposes. The following table illustrates how primary and secondary discourses compare with respect to development, the types of mediation, durability, and ranges of applicability (Table 1).

Table 1

Comparison of Primary and Secondary Discourses

Characteristics	**Primary Discourse**	**Secondary Discourse**
Development	Spontaneous	Through reflection, that is, at meta-level with respect to the primary
Mediation	Predominantly Physical	Predominantly symbolic
Durability	Transient	Lasting
Applicability	Highly Restricted	Universal

(Sfard, 2002)

Discourse and Learning: The MASS System

Gee and Green (1998) offer a framework for discourse analysis for educators in any setting that effectively integrates the key elements of discourse analysis and sociocultural theories of learning and development. The MASS system has four components: *material*, *activity*, *semiotic*, and *sociocultural*. Each of these dimensions of meaning-making can occur in one of two scenarios: (a) situated types of meaning and (b) more abstracted

cultural models. Social languages are distinct from other types of language (i.e., national languages) in that they immediately draw attention to the context and purpose of language use. Gee (1999) compares two language samples that basically convey the same information; yet, have very distinct purposes and thus count as two social languages (p. 27):

1. Experiments show that *Heliconius* butterflies are less likely to ovipost on host plants that possess eggs or egg-like structures. These egg mimics are an unambiguous example of a plant trait evolved in response to a host-restricted group of insect herbivores. (professional journal)
2. *Heliconius* butterflies lay their eggs on *Passiflora* vines. In defense the vines seem to have evolved fake eggs that make it look to the butterflies as if eggs have already been laid on them. (popular science)

Participants were asked to describe the difference between the two social languages. Many would describe sample 1 as being more "academic" or more "scientific." When pushed a little further to identify the discourse markers that index academic or scientific values, some pointed to extra-textual issues such as the genre of the publications (popular science vs. professional journal), thus, the differing discourse communities. Others noted that the language used in sample 1 requires a greater degree of abstraction from the situation. For example, the choice of subject "experiments show" versus "butterflies lay" transforms a single observation into a more generalizable proposition. The lexical choice in sample 1 refers to classes of plants and insects. It is no longer about what a single instance of *Heliconius butterflies* do, but what can be concluded about **all** *Heliconius* butterflies. Some pointed out that there is an unnecessary formality to sample 1 especially when you compare "egg mimics" to "fake eggs." One of the participants compared this example with children's tendency to use informal units of measurement as opposed to formal units of measurement. For example, a child might describe the length of the floor in terms of his or her "red shoes" rather than using more generalizable conventions such as meters, feet, or inches. This might be indicative of the nominalization tendency of mathematics discourse to use nouns rather than adjectives and

nouns (Pimm, 1987; 1995; Morgan, 1998). Sample 1 is also better suited for predicting future behavior which is a value of scientific discourse. Sample 2 is more descriptive and observable and does not require additional inductive reasoning beyond the situation.

Examining the two samples showed not only the linguistic difference between them but also that they represent differentiated learning and thinking (i.e., higher order cognition). Both samples can be considered part of the scientific process with the discursive form of sample 1 representing a more durable and universal type of discourse (secondary discourses). If the importance of discursive identities is considered in learning, the empirical question one might ask is, Which form would a child have more affinity with? This is a critical question for discourse researchers and practitioners because discursive identity, who a person projects themselves to be socially through discourse, is a powerful purveyor of learning and development.

From Language to Discourse Proficiency: The Baseball Language Learners

Using the MASS system as the central unit of analysis for understanding learning and development has four parts:

1. *Material:* The who and what in an interactional frame (the actors, place, social space, time, and objects present (or referred to) during an interaction.
2. *Activity:* What's happening and how is it organized?
3. *Semiotic:* What are they using to make sense and communicate? (This includes gestures, images, or other symbolic systems)
4. *Sociocultural:* What are participants thinking, feeling, and being?

In order to make these ideas more concrete, participants were asked to answer the following questions:

1. What discourses have you partially or fully mastered?
2. Describe features of the discourse that marked membership.
3. Which discourses do you consider "primary" and which ones do you consider "secondary"?

After discussing various discourses and features that marked membership within those communities, I decided to focus the discussion on a typical scenario that is grounded in the baseball discourse community. I divided the participants into three homogenous (self-selected) groups with respect to expertise in that community: the experts, the casual fans, and the "BLLs" (Baseball Language Learners). A list of discrete words and phrases were placed on the board that each group had the task of defining: bat, ball, strike, diamond, base, steal, hit and run, stealing home, batting three hundred, triple crown, run, out, balk, save, and bean ball.

As expected, the expert group and those who consider baseball to be a primary discourse were easily able to define these terms. However, the novice group (our affectionate term "BLLs") struggled to accurately make sense of the terms within a baseball context. The point of the activity was clear as many of the members of this group expressed how for the first time they experienced what it was like to be an English Language Learner (ELL).[6] Of course, they all spoke English, but they didn't speak baseball. As a result, "bat" was more like a bird than a stick, and "ball" was a spherical object instead of a pitch that isn't good to hit, etc. Levinson (1983) argued that it doesn't make sense to talk about any kind of meaning without an activity system that frames meaning. Even apparently discrete meaning-making is predicated on situated and action based participation. The activity system, in this case baseball, is governed by explicit and implicit rules that discourse members know in order to successfully make sense. (This does not necessarily mean they play or are good players, but rather that they are good sense makers within the activity).

The activity system mediates meaning with respect to the other three dimensions of Gee and Green's (1998) framework. There are implications for mathematical problem solving. I gave the following simple arithmetic problem to the participants:

> Barry Bonds, one of the most prolific home run hitters of the modern era, slugged over eight-hundred in one season. If he had six hundred at bats, how many total bases did he get?

This problem is not complicated for someone who is a baseball discourse community member; however, it illustrates how mathematical meaning-making can be situated. All of the

"baseball novices" were stumped by this problem; of course, the experts were able to solve it right away and the homogenous grouping was intended to make this point visible to all the participants rather than a model of "best practice" (although it made the point in favor of heterogeneous grouping of language learners). In fact, simple, straightforward and seemingly universal numerical representations like "hundred" have two different meanings within the same question stem. The first instance "eight-hundred" represents a percentage where the whole is not referred to as 100% but rather 1000%. The second instance of "hundred" is the more accustomed usage (the value 100). As shown below, the language load of the math problem can be virtually eliminated by providing the formula for slugging percentage, and hence anyone with the knowledge of how to employ formulas could derive the answers (although "eight hundred" might still be a stumbling block).

> Barry Bonds, one of the most prolific home run hitters of the modern era, **slugged** over **eight-hundred** in one season. If he had six **hundred** at bats, how many total **bases** did he get?
>
> Slugging Percentage=Total Bases/At Bats
>
> 1. Total Bases/At Bats=.800
> 2. Total Bases/600=.800
> 3. Total Bases=600*.800
> 4. =480

However, this type of modification presumes math to be free from linguistic and discursive issues and does not always work, especially in high-stakes mathematical assessments.

One of the school participants, who was a doctoral fellow at the time and is now a mathematics teacher educator, thought that this type of activity would be ideal to use in a mathematics methods course. After the conclusion of the session, she reflected upon the BLL activity,

> I think this would be a great example to use with the preservice teachers to have them get in the shoes of those ELLs who have acquired conversational fluency in English but not academic—mathematical—fluency in English. Most people, including teachers, tend to think of ELLs as those who have difficulty speaking in English or have a heavy foreign accent. If a child speaks English fluently or has a native-like American-English accent then, in their minds, that child is not an ELL.

The activities that are typically used with (monolingual) preservice teachers to have them experience what ELLs experience in the classroom, and to perhaps model strategies that can be used to accommodate ELLs are often in a language that none of the preservice teachers speak. Such activities, for example, include a mathematics problem written in a language the preservice teachers are not familiar with, or a health video giving instructions in Farsi (Harding-DeKam, 2007). While these activities can be useful to have preservice teachers experience what it feels like to be an ELL who has recently moved to the U.S. and speaks no English, the majority of the ELLs that preservice teachers will be teaching will not fall into that category. In fact, most ELLs have some level of conversational fluency in English, and many of them might not have an easily detectable foreign accent, making it difficult for teachers to classify them appropriately as ELLs. According to Cummins (1981) conversational fluency in English is acquired within 2 years, while it takes 5 to 7 years to acquire academic (including mathematical) fluency in English. Teachers need to be aware of this important distinction, and they need to understand its implications for teaching mathematics to ELLs. Preservice teachers are often taught this distinction in their coursework but do not necessarily make connections with what this means for teaching mathematics to ELLs (Vomvoridi-Ivanovic & Khisty, 2007).

Conclusion

In this article, I provided conceptual and methodological tools as well as activities that can be used for the preparation and professional development of both mathematics teacher educators and mathematics teachers to aid their development of a more situated and social view of mathematical discourse and its relationship to student learning, particularly how mathematical discourse relates to LMS. The concrete examples discussed in this article help make the discursive nature of mathematics more overt for those who believe that mathematics is a universal language. As the field considers the mathematics education of LMS, mathematics teacher educators as well as mathematics teachers can draw on the notions of primary and secondary discourses to

move beyond static views of development, especially *vis a vis* mathematics learning.

To improve the mathematics education of LMS, mathematics teacher educators should receive professional development that supports them in including issues of language and discourse in their mathematics teacher preparation courses and in professional development settings with in-service mathematics teachers. This, in turn, will help mathematics teachers begin to develop knowledge that is required to support the mathematics learning of LMS. Teacher educators need more research that examines what preservice teachers learn when they participate in activities designed to build critical awareness about issues in language learning and develop an emic perspective of the challenges encountered by ELLs and other members of non-dominant populations who engage in non-orthodox forms of mathematical meaning-making (e.g., Saxe, 1988). Although the activities presented in this article have great potential to move preservice teachers towards these critical understandings of discourse, language, and learning, it is important for teacher educators to develop new activities that are suited to the needs of their preservice teachers.

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[1] "Language" refers to the structural aspects of language (i.e., code) and/or the use of national languages (e.g., Spanish, English). "Discourse" refers to the specialized and situated language of mathematics (e.g., quantitative and symbolic language). The distinction between "language" and "discourse" will be elaborated later in the paper.

[2] The [ ] are a transcription convention used to indicate overlapping talk; colons (:::) indicate prolongation of sound. All names are pseudonyms.

[3] Footing refers to how the mode and frame of a conversation is determined by participants in an interaction, and how speakers empower and/or disempower each other through various linguistic practices that invoke power relations, social status, and legitimacy.

[4] Intersubjectivity is an interdisciplinary term used to describe the agreement between speakers on a given set of meanings, definitions, ideas, feelings, and social relations. The degree of agreement could be partial or sometimes divergent as in the case of deception, sarcasm, irony, or lying.

[5] Cultural Historical Activity Theory (CHAT) is a more recent term used by neo-Vygotskians to emphasize the historical dimensions of learning (e.g., Rogoff, 1995; Sfard, 2002).

[6] English Language Learner (ELL) is a subgroup of Language Minority Students (LMS). It is the common term used in U.S. public schools to classify students for whom English is either their second language or come from bilingual homes.

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The Devalued Student: Misalignment of Current Mathematics Knowledge and Level of Instruction

Steven D. LeMire, Marcella L. Melby , Anne M. Haskins , and Tony Williams

Within this study, we investigated the association between 10th-grade students' mathematics performance and their feelings of instructional misalignment between their current mathematics knowledge and educator support. Data from the 2002 Education Longitudinal Study, which included a national sample of 750 public and private high schools in the United States, was used for the investigation. Our findings indicate that student perceptions of both instructional alignment and educator support are associated with mathematics performance. Students who reported receiving misaligned instruction in mathematics and felt devalued by educators had lower mathematics performance than students who reported aligned mathematics instruction and who felt valued by teachers. A key implication for practitioners of this work is that mathematics educators should consider cognitive and affective elements of student development. Specifically in addition to cognitive factors, the affective elements of student capacity to receive, respond to, and value whole-group mathematics instruction in academically diverse classrooms should be considered in curriculum planning.

> *Learning is not just the acquisition and manipulation of content; how and how well we learn is influenced by the affective realm – our emotions and feelings – as well as by the cognitive domain.* (Ferro, 1993, p. 25)

It is well known that not all students reach their full mathematics potential in high school. According to Tomlinson et

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