Research on the Problem of Translating Natural Language Sentences into Algebra

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Abstract

The author conducted a study to determine how a competent person translates a natural language sentence into an algebraic equation. A current theory of how translation occurs is espoused by a group of cognitive scientists who propose that the competent translator uses only conceptual strategies in the translation process. I propose and investigate an alternate theory that the translation skills of a competent student are based on a syntactic analysis and syntactic manipulation of the sentence. In most cases such direct, syntactic translation processes will suffice, but for a small minority of sentence types a decision must be made to abandon the syntactic processes and use conceptual strategies. A detailed syntactic model which I developed includes this decision process. The theory was tested by embedding the two alternate theories in classroom treatments using a repeated measures control group design.

Subjects included five college algebra classes. Two groups were taught the translation process as outlined in the syntactic theory, whereas another group was taught the translation process stressing conceptual strategies. A pretest was given prior to instruction, a posttest one week after a three day treatment period, and a retention test twelve weeks after the posttest.

Results indicated that the students who received the syntactic treatments had significantly better mean scores on translation tasks on the posttest and on the retention test than students who received either the conceptual treatment or no treatment. On the retention test, the conceptual treatment students' scores decreased so that they were no longer significantly different from those of the control group.

These results suggest that the knowledge components stressed in the syntactic treatments more closely parallel the knowledge components of a competent translator. Thus, this study provides evidence that competence in translation of a natural language sentence into an algebraic equation involves a syntactic manipulation of the sentence, and that instruction that includes syntactic strategies has pedagogical merit.

Annotated Bibliography

Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education, 13* (1), 16-30.

A study with 150 students detected the reversal error and blamed it on two types of misconceptions. The first was word order matching (syntactic) and the second was a static comparison approach (semantic). Successful students used an operative approach, and it was suggested that talk-aloud protocols might help in designing better instructional strategies.

Clement, J., Lochhead, J., & Monk, G. (1981). Translation difficulties in learning mathematics. *American Mathematical Monthly, 88,* 286-290.

This is the original article concerning the Student/Professor problem. The authors suggested that the physical representation of a relation is an important first step in the translation process.

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Clement, J., Narode, R., & Rosnick, P. (1981). Intuitive misconceptions in algebra as a source of math anxiety. *Focus on Learning Problems in Mathematics, 3* (4), 36-45.

A study with 15 students. Even though the students were found to have a clear conception of the problem they still made reversal errors. The authors suggested that the error is resilient, and unless the students discover why the approach does not work they are unlikely to abandon it.

Cooper, M. (1986). The dependence of multiplicative reversal on equation format. *Journal of Mathematical Behavior, 5*, 115-120.

A study with nearly a thousand high school students who were asked to write statements for the equations to represent relations between a number of dogs and cats, as given by $C = 4D$, $C = 4 \text{ x } D$, $x = 4y$, and $x = 4 \text{ x } y$. Results suggested that the insertion of the multiplication sign (x) lowers the reversal error, while the use of different letters as variables makes no difference.

Davis, R. (1980). The postulation of certain specific, explicit, commonly-shared frames. *Journal of Mathematical Behavior, 3* (1), 167-201.

An excellent postulation of representation structures called *frames*. The retrieval of a *label frame* instead of a *numerical-variables equation frame* is suggested as a possible cause of certain reversal errors.

Fisher, K. (1988). The students-and-professors problem revisited. *Journal for Research in Mathematics Education, 19* (3), 260-262.

An experiment with 58 students in which 30 were given the *S-P* version of the student-professor problem and 28 were told to use N_s and N_p. The more specific notation seemed to hinder the students, and a conclusion was that the error is not a literal mapping error—but a more deep seated one.

Gray, E. (1992). An analysis of syntactic skills used in translating natural language sentences into algebraic equations. *Dissertation Abstract International, 53*, p. 433-A (University Microfilms No. 92-19,537)

A study with four college algebra classes developed and tested a theory about competence in translation. It finds a significant difference in the scores of students given a syntactical treatment over those given a semantic treatment.

Kaput, J., & Sims-Knight, J. (1983). Errors in translations to algebraic equations: Roots and implications. *Focus on Learning Problems in Mathematics, 5 (3)*, 63-78.

A description of two studies. The first used 266 third and fourth year secondary students and suggested that sentences utilizing a natural representational system are more difficult to translate. The second used 147 heterogeneous university students and 138 engineering freshmen to test a hypothesis that good proportional reasoning and a good understanding of the concept of variable are necessary skills for translating sentences into equations.

Kaput, J., Sims-Knight, J., & Clement, J. (1985). Behavioral objectives: A response to Wollman. *Journal for Research in Mathematics Education, 16*, 56-62.

A critique of the Wollman (1983) study in which three concerns were listed: 1) no cognitive models were used to explain the given phenomena, 2) no discussion of other possible knowledge structures, and 3) no description of a development of relevant concepts. It was suggested that such attempts might correct translation errors without improving the curriculum.

Kirshner, D., McDonald, J., Awtry, T., & Gray, E. (1991). The cognitivist caricature of mathematical thinking: The case of the students and professors problem. In R. Underhill (Ed.), *Proceedings of the Thirteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Blacksburg, VA: PME-NA.

A study in a university mathematics department using twenty professors, five instructors, and seventeen graduate students which tested the syntactic theory of translation described by Gray (1992). There was a response-time difference in translation of different types of sentences and there was a high frequency of reversal errors in this group.

Lochhead, J. (1980). Faculty interpretations of simple algebraic statements: The professors' side of the equation. *Journal of Mathematical Behavior, 3* (1), 29-38.

In this study 200 university faculty and 150 high school students were asked to write a sentence for the equation $A = 7S$ (A is the number of assemblers in a factory and S is the number of solderers in a factory). The dismal results suggested more emphasis on developing translation skills.

MacGregor, M. (1990). Writing in natural language helps students construct algebraic equations. *Mathematics Education Research Journal, 2* (2), 1-11.

A study with 158 first year university students was conducted to discover whether the translation from graphical data to equation was affected by first writing the data as a sentence. The writing was helpful and need not be in correct algebraic syntax order. It is suggested that more studies should try to determine the extent of talking and writing about mathematics as an influence on students' learning.

MacGregor, M., & Stacey, K. (1993). Cognitive models underlying students' formulation of simple linear equations. *Journal for Research in Mathematics Education, 24* (3), 217-232.

Data collected from 281 students in the ninth grade do not support the often used suggestion that students frequently use syntactic translation. The study postulates that the typical cognitive model uses syntactic, semantic and other types of processing, and that more research should look at how a student links a mathematical situation and its formal algebraic description.

Rosnick, P., & Clement, J. (1980). Learning without understanding: The effect of tutoring strategies on algebra misconceptions. *The Journal of Mathematical Behavior, 3* (1), 3-28.

A study using nine students in an hour-long instructional intervention to stop reversal errors. They suggest that writing a correct equation does not necessarily always imply understanding and the concepts of variable and equation should not be treated lightly.

Wollman, W. (1983). Determining the sources of error in a translation from sentence to equation. *Journal for Research in Mathematics Education, 14* (3), 169-181.

This study used eight classes of female elementary education majors to study the significance of reversal errors. The four components were a) understanding the natural language sentence, b) understanding an algebraic equation of the type $y = kx$, c) a method for generating the equation from the sentence, and d) a method for checking. Checking was found to be the most significant component of success.